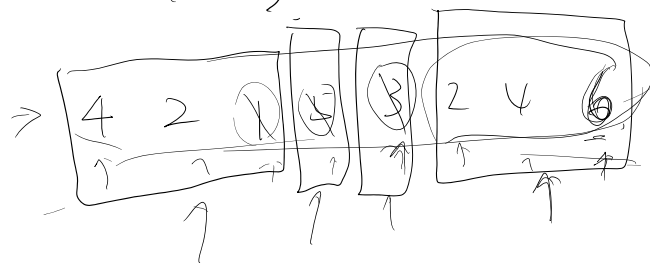


$$E(A | B) = 1.5$$

$$= \sum_{i=1}^{\infty} i \cdot \Pr(A=i | B)$$

$$= \frac{\sum_{i=1}^{\infty} i \cdot \Pr(A=i \wedge B)}{\Pr(B)}$$

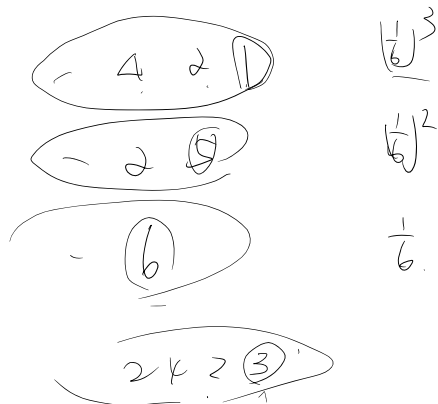


$$E(5) \approx E(3)$$

1, 3, 5, 6

$$\frac{3!}{4!} = \frac{1}{4}$$

$$E(\text{首次掷出 } 1/3/5/6 \text{ 的步数} | \text{之前只掷出偶数}) = 1.5$$

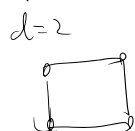


$$2^{\circ} E(\text{首次 } 6 \text{ 的步数} | \text{掷出 } 6 \text{ 之前的数字是不下降的})$$

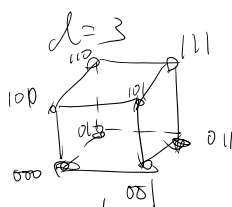
routing problem.

hypercube

dim-d



2^d nodes. $= N$



$$\text{degree} = d \sim \log N$$

$$\text{max. shortest path} = d \sim \log N.$$

$$\# \text{ edge} = 2^d \times d / 2 = d \times 2^{d-1}$$

$$(x_1, \dots, x_d) \rightsquigarrow (y_1, \dots, y_d)$$

$$\begin{array}{ccc} (1000) & \rightsquigarrow & (0001) \\ \downarrow & \nearrow & \\ (0000) & & \end{array}$$

$$\sqrt{\frac{N}{d}} = \sqrt{\frac{2^d}{d}} = \frac{2^{\frac{d}{2}}}{\sqrt{d}}$$

Instance 1: $(x_1, \dots, x_d) \rightsquigarrow (1-x_1, 1-x_2, \dots, 1-x_d)$ good instance.

$\downarrow \quad \uparrow \quad \uparrow$

$(1-x_1, x_2, \dots, x_d) \rightsquigarrow (1-x_1, 1-x_2, x_3, \dots, x_d)$

$$\frac{2^d \times d}{2^{d-1} \times d} \quad e = (z_1, \dots, \underbrace{z_i}_{\text{package}}, \dots, z_n) \rightsquigarrow (z_1, \dots, z_{i-1}, 1-z_i, z_{i+1}, \dots, z_n)$$

$2 \uparrow$

$$(1-z_1, 1-z_2, \dots, 1-z_{i-1}, \underbrace{z_i}_{\text{package}}, z_{i+1}, \dots, z_n)$$

$$(z_1, z_2, \dots, z_{i-1}, 1-z_i, 1-z_{i+1}, \dots, 1-z_n)$$

Instance 2: $(x, y) \rightsquigarrow (y, x)$

$$\begin{array}{cc} \uparrow & \downarrow \\ \frac{d}{2} \text{位} & \frac{d}{2} \text{位} \\ \downarrow & \downarrow \\ d \text{位} & d \text{位} \end{array}$$

$$(x, 0) \rightsquigarrow (0, x)$$

$$e \quad (0, 0) \rightsquigarrow (0, 0)$$

$\uparrow \quad \uparrow$

$\frac{d}{2} \text{位} \quad \frac{d}{2} \text{位}$

$$\# \text{ packets: } 2^{\frac{d}{2}-1}$$

$$\frac{2^{\frac{d}{2}}}{\sqrt{d}}$$

$$(x, \dots, x) \rightsquigarrow (00 \mid 000) \rightsquigarrow (000, 000)$$

$$\begin{array}{c} \underbrace{(101, 000)}_{\alpha} \rightarrow \underbrace{(001, 000)}_{\alpha} \xrightarrow{e} (000, 000) \\ \downarrow \\ (000, 101) \leftarrow (000, 100) \end{array}$$

randomized bit fixing algo

$$(x, y) \rightsquigarrow (y, x)$$

$$(101, 000) \rightarrow (100, 000) \rightarrow (100100) \rightarrow (100101) \rightarrow (000101)$$

$$e: \begin{array}{ccc} \vec{0} & | & \vec{0} \\ \uparrow & & \uparrow \\ d/2 - 1 & & d/2 \end{array} \rightarrow (\vec{0}, \vec{0})$$

$$\underbrace{(\vec{x}, \vec{0}) \rightsquigarrow (\vec{0}, \vec{x})}$$

$$|x| = c \quad \Pr(\text{exists } e)$$

$$= \frac{c! (c+1)!}{(2c+2)!}$$

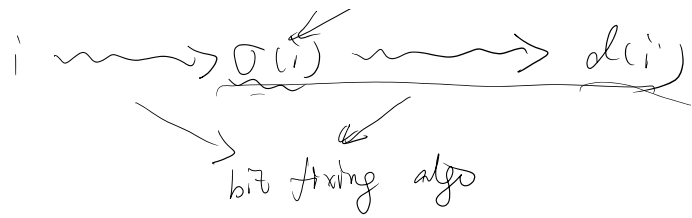
$$\Theta = \sum_{c=0}^{d/2-1} \frac{c! (c+1)!}{(2c+2)!} \times \frac{\#\{x \mid |x|=c\}}{\binom{d/2-1}{c}}$$

$$\text{for } c = \frac{d}{2} - 1 \quad \frac{(\frac{d}{2}-1)! (\frac{d}{2})!}{(\frac{d}{2})!} \binom{\frac{d}{2}-1}{\frac{d}{2}-1} = \frac{\Omega(d)}{2}$$

Two-phase randomized bit fixing algo.

$$i \rightsquigarrow d(i), \quad \text{randomly chosen (independent)}$$

$$i \rightsquigarrow \underline{0(i)} \rightsquigarrow d(i)$$



$$i \rightsquigarrow \sigma(i)$$

$$x \rightsquigarrow \sigma(x)$$

$$\underline{e} = (\vec{a}, c, \vec{b})$$

$$y \rightsquigarrow \sigma(y)$$

$$\begin{matrix} k & 1 & d-k-1 \\ \downarrow & \downarrow & \downarrow \\ \vec{a} & 1-c & \vec{b} \end{matrix}$$

$$y: \vec{a} \quad c \quad \vec{b}$$

$$\sigma(y): \vec{a} \quad 1-c \quad \vec{b}$$

$$\Pr(\sigma(y) = \vec{a} \quad 1-c \quad \vec{b}) = \frac{1}{2^{b+1}}$$

$$\#\{y \mid y = \vec{a} \quad c \quad \vec{b}\} = 2^k$$

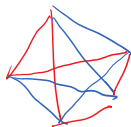
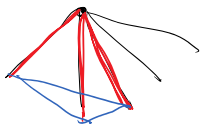
$$\mathbb{E}(\#\{y \rightsquigarrow \sigma(y) \mid \sigma(y) = \underline{e}\}) = 2^k \times \frac{1}{2^{b+1}} = \frac{1}{2}$$

$$\mathbb{E}(\text{delay}(x \rightsquigarrow \sigma(x))) \leq d \times \mathbb{E}(\#\{y \rightsquigarrow \sigma(y) \mid \sigma(y) = \underline{e}\})$$

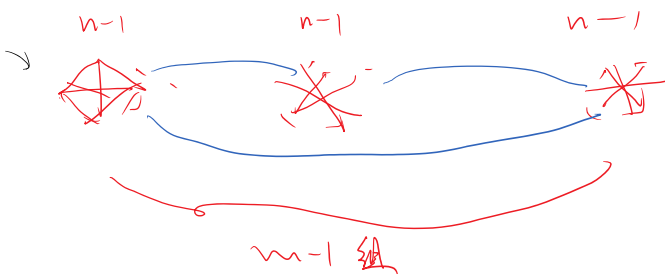
$$= \frac{d}{2}$$

Ramsey number.

$$R(3,3) = 6$$



$$R(m,n) \geq (m-1)(n-1) + 1$$



k 个边.

G is good : 没有同色 K_n .

$$R(n, n) = k$$

$$\Pr(G \text{ is good}) > 0$$

$$\Pr(G \text{ is not good}) \leq \binom{k}{n} \cdot \frac{1}{2^{\binom{n}{2}-1}}$$

$$\leq \left(\frac{ek}{n}\right)^n \cdot \frac{1}{2^{\frac{n^2}{2}-\frac{n}{2}-1}}$$

$$\text{取 } k = 2^{\frac{n}{2}}$$

$$= \frac{e^n \cdot 2^{\frac{n}{2}+1}}{n^n} < 1$$

$$\Pr(G \text{ is good}) > 0$$

$$R(n, n) > 2^{\frac{n}{2}}$$

Max cut.

$$E(\text{\#edge in cut})$$

$$X_1, \dots, X_m$$

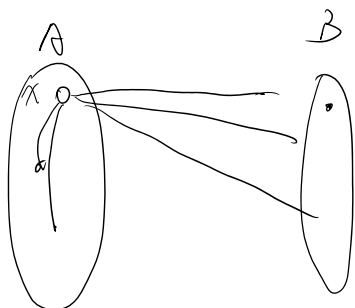
$$= E(\sum X_i)$$

$$X_i = \begin{cases} 1 & \text{1-st edge in cut} \\ 0 & \end{cases}$$

$$= \sum E(X_i)$$

$$= \frac{m}{2}$$

construction:



$$x \in A.$$

$$\{y \mid y \in B \wedge (x, y) \in e\}$$

$$\{y \mid y \in A \wedge (x, y) \in e\}$$